

IDIOSYNCRATIC VOLATILITY

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Abstract. Identifying stock returns' idiosyncratic volatility (IV) is essential for asset pricing, portfolio construction, and empirical studies. IV plays a central role in explaining the cross-section of returns, including "IV puzzle" and designing investment strategies. The existing literature defines IV as the difference between a stock's total return volatility and the portion explained by (linear beta) pricing models. We show that this conventional measure is fundamentally misspecified because in reality no traded asset has zero-variance returns. We identify the true IV measure and demonstrate that the ratio of the true to the literature's misspecified IV measure lies in the interval $[0,1]$. This implies measurement-error magnitudes ranging from zero to infinity, where zero misspecification occurs with probability zero. The mismeasurement is arbitrary—nonmonotonic and nonlinear in any stock or market parameter. These results suggest that, under the literature's definition of IV, no econometric method can correct for the misspecification. Empirical studies relying on this conventional IV measure are arbitrarily distorted.

Key Words: Idiosyncratic volatility, Misspecification, Linear beta pricing, Zero beta pricing

JEL Codes: G12, G19

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1 Introduction

Identifying stock returns' idiosyncratic volatility (IV) is essential and has been a subject of intensive theoretical and empirical research [Merton (1987), Lehmann (1990), for example]. Following the introduction of an "Idiosyncratic Volatility Puzzle" [Ang, Hodrick, Xing, and Zhang (2006)], IV research intensified.¹ IV has become central to explaining cross-sectional returns and designing investment strategies. The literature identifies IV as the difference between stock returns' total volatility and its priced portion, typically using linear beta pricing models [e.g., Fama and French (1992, 1993), and followers].

Within the standard mean-variance framework, also known as Markowitz's world, this identification relies on a counterfactual premise: the existence of a traded asset with zero-variance returns. As Black (1972) emphasized in introducing the zero-beta Capital Asset Pricing Model (CAPM), such an asset does not exist in the real world, and allowing for it can lead to mispricing. Building on this insight, we show below that the literature's IV measure is fundamentally misspecified.

We show that in the absence of a "zero-variance" asset, risky assets' return volatilities consist of three mutually uncorrelated components: *i.* idiosyncratic volatility, which is nonsystematic, diversifiable, and not priced, *ii.* zero beta volatility, which is systematic, not diversifiable, and not priced, and *iii.* market portfolio volatility (briefly, market volatility), which is systematic, not diversifiable, and priced. While the true IV is component *i.* alone, the literature identifies IV as the sum of components *i.* and *ii.*, conflating the true IV with systematic, nondiversifiable, zero-beta volatility.

¹ Recent studies have expanded the analysis of IV-based return predictability beyond equities, examining markets such as options and corporate bonds [Cao and Han (2013), Zhan, Han, Cao, and Tong (2022), Chen, Roussanov, Wang, and Zou (2025)].

Under the correct IV identification, we show that the ratio of a stock's true IV to the literature's misspecified one takes values in the interval $[0.1]$.² This implies measurement-error magnitudes ranging from zero to infinity, where zero misspecification occurs with probability zero. The mismeasurement is arbitrary—nonmonotonic and nonlinear in any stock or market parameter, such as stock returns means, variances, true IVs, excess returns over the market portfolio, portfolio frontier curvature, excess return of the market portfolio over the global minimum variance portfolio (GMVP), excess return of the GMVP over the zero beta portfolio, market portfolio volatility, and zero beta portfolio volatility. These results suggest that, under the literature's definition of IV, no econometric method can correct for the misspecification. Consequently, empirical studies relying on this conventional IV measure, including the IV puzzle, may be arbitrarily distorted.

The purpose of this study is to provide a correct IV identification. We leave for future studies the consequences of using the correct IV identification rather than the literature's misspecified IV measure. The IV puzzle, for example, can be mitigated or enhanced under the correct IV measure.

Our analysis draws on classic portfolio theory and linear beta pricing, due to Markowitz (1952), Sharpe (1963, 1964), Lintner (1965), Mossin (1966), and Merton (1972). Black (1972) introduced zero beta CAPM, a linear beta pricing with no zero-variance asset. Black showed that, while the classical CAPM is a single-index (one-factor) model where Capital Market Line (CML) portfolios are attainable, the zero beta CAPM is a two-index (two-factor) model where only risky portfolios on and within the portfolio frontier are attainable.³ The classical CAPM's CML is the pricing line, but because it is not attainable for investors, it becomes an “imaginary”

² If we define the misspecification ratio to be the misspecified IV to the true one (inverse of the current one), it would be on the interval $[0, \infty]$

³ An intuition behind the zero beta CAPM pricing is that the pricing mathematics is not sensitive to whether a zero-variance asset is traded.

line. The “Efficient Frontier,”⁴ which under the classical CAPM is the CML, becomes the upper, positively sloping lobe of the portfolio frontiers’ hyperbola in the mean-standard deviation cartesian space.⁵

Ross (1976) and followers demonstrated that linear factor structures that span returns can be rotated to have a single-priced factor (possibly a combination of factors perfectly correlated with the pricing kernel), and non-priced (zero beta) factors. Thus, Blacks’ zero beta CAPM becomes the simplest pricing structure, consisting of a single-priced factor and a single non-priced (zero beta) one, a structure that can represent multi-factor structures. We demonstrate our results within this structure.

This property corresponds to the Tobin (1958) analysis, which implies a “Two Funds Separation” property. See also Huang and Litzenberger (1988, Section 1.27 and Chapter 4). The property suggests that an investor’s optimal portfolio, regardless of his/her risk-aversion level, is a combination of two funds. As the Two Fund Separation property holds under the classical CAPM and the zero beta CAPM, two distinct CML/portfolio frontier portfolios (funds) are sufficient to span the CML/portfolio frontier.

The intuition behind our results is as follows. In Markowitz’s world of two or more risky assets, the return of each asset can be replicated (spanned) by a combination of three assets. First, form a combination of two frontier portfolios, the (priced) market portfolio and its (unpriced) zero beta portfolio, such that the resulting portfolio matches the asset’s expected return. As both portfolios are portfolio frontier portfolios, their combination is a frontier portfolio [see, for example, Huang and Litzenberger (1988, Chapter 4)]. We add a third asset representing noise, or idiosyncratic volatility, whose return is zero mean and uncorrelated with

⁴ The locus of portfolio frontier portfolios that are not mean variance dominated.

⁵ The mean is on the vertical axis.

the returns of the first two assets. The volatility of the third asset (idiosyncratic volatility) complements the volatility of the first two assets, to equal the volatility of the replicated asset.

Because the returns of the three replicating assets are mutually orthogonal (uncorrelated), the volatility of the combination of the three replicating assets is a weighted sum of their three volatilities.

We thus decompose the replicated asset's volatility into three components: *i.* idiosyncratic volatility, which is nonsystematic, diversifiable, and not priced, *ii.* zero beta volatility, which is systematic, not diversifiable, and not priced, and *iii.* market volatility, which is systematic, not diversifiable, and priced.

The literature identifies an asset's idiosyncratic volatility as its total volatility minus the third component or, equivalently, as the sum of the first and second components. Empirical studies estimate idiosyncratic volatility as the variance of residuals from factor models. The Fama and French (1993) three-factor model has long served as a benchmark, alongside the CAPM or statistical models such as principal components analysis [e.g., Ang, Hodrick, Xing, and Zhang (2006), Herskovic et al. (2016)].⁶ More recent studies extend this approach using the Fama and French (2015) five-factor model or the six-factor model that includes Carhart's (1997) momentum factor [Qadan (2019), Qadan and Shuval (2022), Renault, Van Der Heijden, and Werker (2023), Bali and Weigert (2024), Sezgin, Doruk, Barak, and Ertuğrul (2025)].⁷ All these studies estimate idiosyncratic volatility from residuals orthogonalized to pricing factors. Thus, they conflate truly diversifiable idiosyncratic risk with systematic but nondiversifiable zero-beta risk, leading to a misspecification in the measurement of idiosyncratic volatility.

⁶ Several studies estimate idiosyncratic volatility using the market model. Campbell, Lettau, Malkiel, and Xu (2001) propose a method to estimate aggregate idiosyncratic volatility orthogonal to market risk without directly estimating the market model. Durnev, Morck, Yeung, and Zarowin (2003) estimate firm-level idiosyncratic volatility from the residuals of regressions of individual stock returns on both market and industry returns.

⁷ A strand of literature estimates the conditional idiosyncratic volatility to capture its time-varying dynamics and investigate its relationship with expected stock returns (Fu, 2009; Guo, Kassa, and Ferguson, 2014; Liu, 2022).

We note that while the current IV literature chooses to investigate capital markets with a traded zero-variance asset, essential asset pricing literature has long adopted Black's (1972) view of no traded zero-variance asset [see, for example, Roll and Ross (1994) and Kandel and Stambaugh (1995)].

Section 2 provides theoretical identification and total volatility decomposition. Section 3 offers empirical demonstrations. Section 4 concludes.

2 The Model

We will use these notational conventions: constants and variables will appear in italic (slanted) font, operators and functions in straight font, and vectors and matrices in boldface (dark) straight font.

Assume a market with N risky securities. Let R_i , $i = 1, \dots, N$, be the random rates of return of the risky security i . We do not specify the probability distributions of the rates of return but assume means and variances that are real finite numbers. For simplicity, we assume no redundant securities.

Following Markowitz (1952), Merton (1972), Black (1972), and also Huang and Litzenberger (1988), Feldman and Reisman (2003), and Feldman and Diacogiannis (2013), standard results are the existence, in our return space, of the portfolio frontier, the CML, the market portfolio, the “intercept” of the tangent to the portfolio frontier at the market portfolio at the zero beta rate, and the zero beta portfolio to the market portfolio which is a portfolio frontier portfolio of the expected return of the “intercept.”

The notations we use are standard; for simplicity and brevity, we skip some of them. definitions used in the figures below. $\mathbf{E}$ is the expectation operator, $Z\beta_M$ is the zero-beta portfolio to the market portfolio, σ is standard deviation, $\sigma_{Z\beta_M}$ is the standard deviation of portfolio $Z\beta_M$, MV is volatility generated by the market portfolio, $Z\beta_M V$ is volatility generated

by the zero beta portfolio, and IV is idiosyncratic volatility. We also use trivial modifications of these notations.

Figure 1 below depicts the returns space, the portfolio frontier, the market portfolio, the CML, the “intercept,” the zero-beta portfolio to the market portfolio, and volatility decomposition of each of the three assets. This volatility decomposition is on the horizontal line from assets’ volatilities to the zero-volatility vertical axis.

- Market portfolio – we call the volatility generated by the (potentially leveraged) market portfolio, MV . In the return space, this volatility falls exclusively to the left of the CML, as the market portfolio spans the CML and the volatility is measured from left to right.
- The zero-beta portfolio to the market portfolio – we call the volatility generated by the (potentially leveraged) zero-beta portfolio to the market portfolio, $Z\beta_M V$. In the returns space, this volatility falls exclusively to the right of the CML, as the zero-beta portfolio to the market portfolio is not part of the CML
- A typical risky asset – The volatility has three components. The first is the interval from the asset’s total volatility point to the portfolio frontier. By construction, this volatility is diversifiable, and we call it idiosyncratic volatility. The second component is the interval from portfolio frontier to the CML. This volatility is systematic, coming from the zero-beta portfolio, and is not diversifiable. It is not priced as the zero -beta return is uncorrelated with the market portfolio’s return. The third component is the volatility generated by the market portfolio, which we call market volatility. It is systematic, nondiversifiable and priced.

We note that the volatility of Figure 1 below depicts the volatility decompositions described above and identifies the true and misspecified IV components.⁸

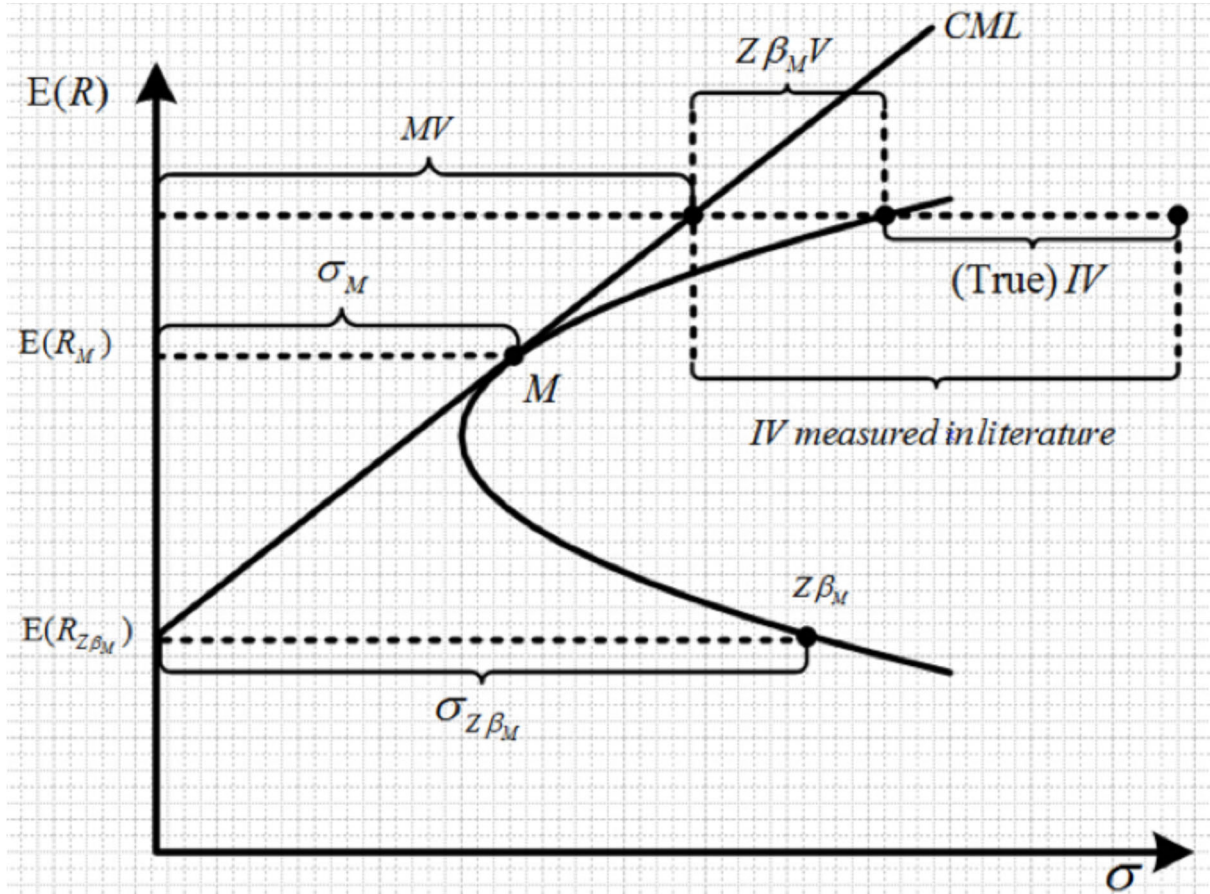


Figure 1: The returns space, portfolio frontier, CML, “intercept,” zero beta portfolio, and total volatility decompositions for three assets

Figure 2 below is like Figure 1, except for a highly different ratio of market volatility and zero-beta volatility. It demonstrates how seemingly unrelated properties of the return space substantially affect the misspecification.

⁸ A portfolio’s volatility components can be easily calculated as follows: *i.* the total volatility of a certain portfolio is given exogenously *ii.* to determine the true IV, the volatility of the corresponding portfolio (of the same expected return) on the portfolio frontier can be calculated, say, by using Feldman and Reisman (2003), *iii.* the volatility of the corresponding portfolio (of the same expected return) on the CML can be calculated using the CML equation.

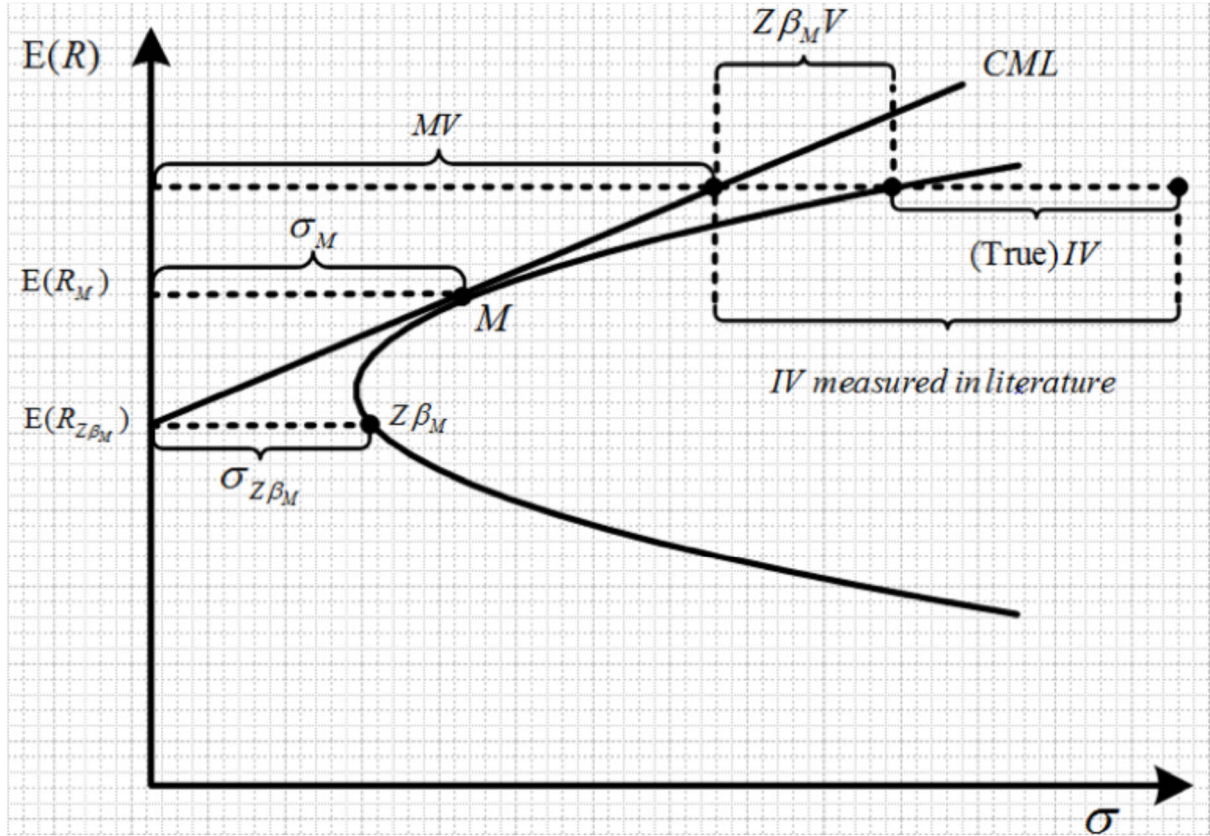


Figure 2: The decomposition of total volatility with different ratios of pricing benchmark (market portfolio) variance to $Z\beta_M V$

Figure 3 below demonstrates various levels of misspecification as functions of assets' level of expected returns and of idiosyncratic volatility, and demonstrates high codependence of these attributes.

Assets with expected returns of the market portfolio (a set of “measure zero”) have no zero-beta volatility, thus generating no IV misspecification, and a ratio of true to misspecified IV of 1. Portfolio frontier assets—thus with no IV—at expected returns different from that of the market portfolio generate a ratio of true to misspecified IV of 0 (also a set of “measure zero”).

The nonlinearity and nonmonotonicity of zero beta volatilities in expected returns induce nonlinearity and nonmonotonicity in the true to misspecified IV ratios. Starting from higher expected returns, going down, the distance between the CML and the portfolio frontier

diminishes from infinity to zero, at the market portfolio's expected return, then increases to infinity as expected returns diminish. The nonlinearity is a consequence of the portfolio frontier hyperbola's curvature.

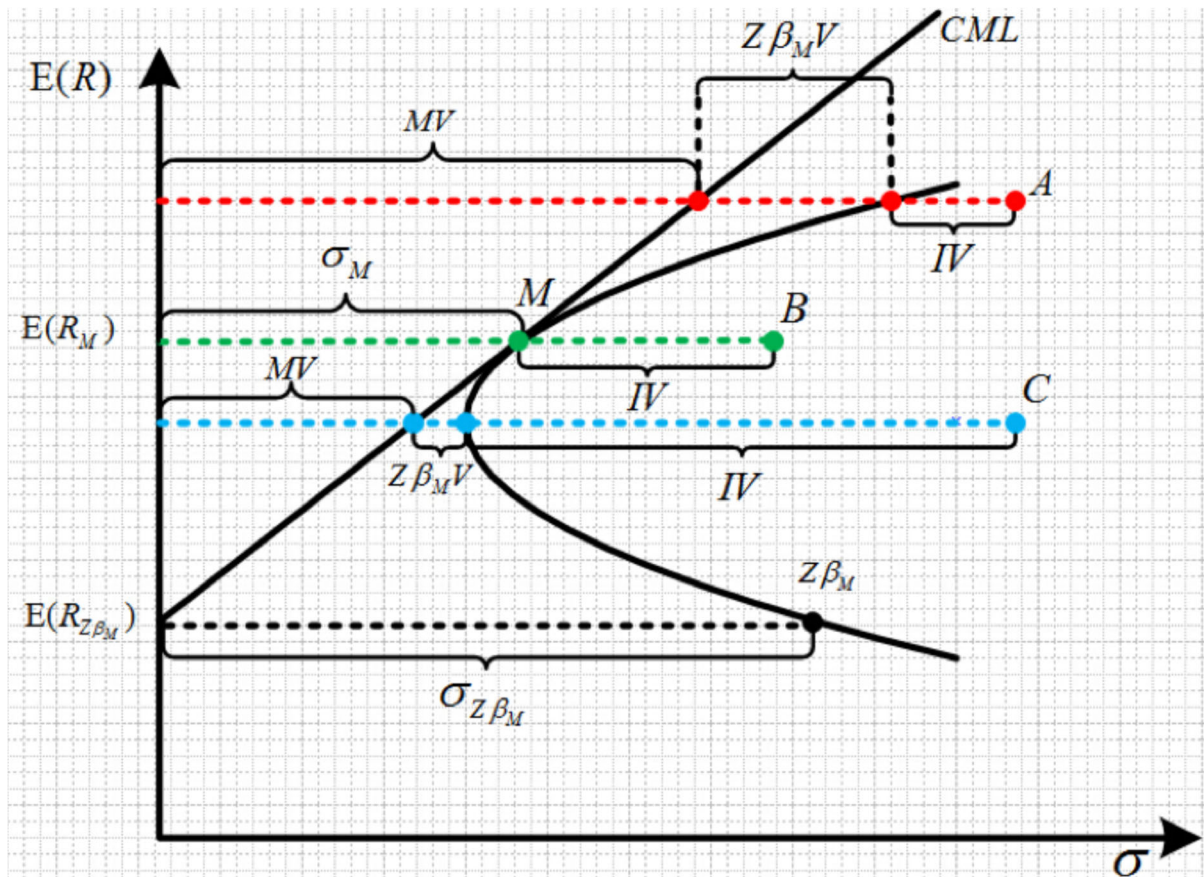
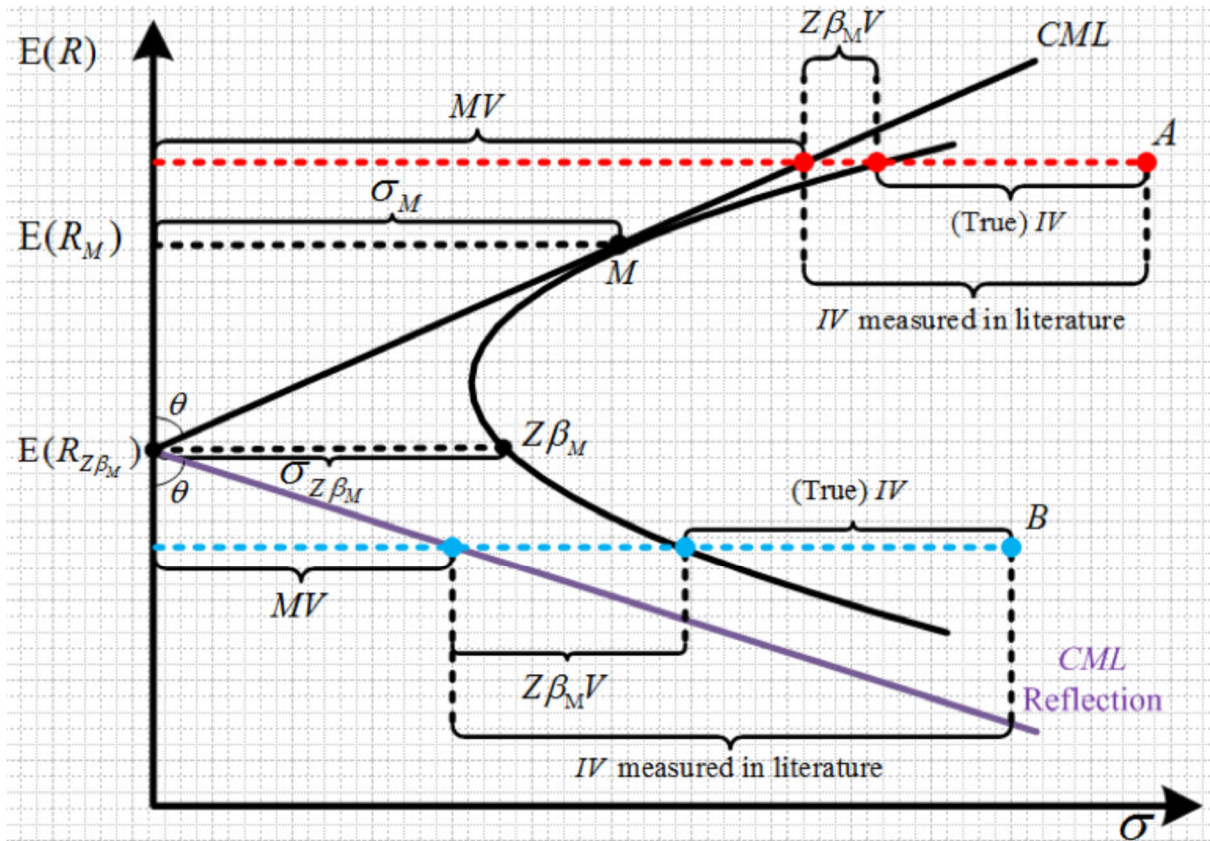


Figure 3: Demonstration of misspecification levels

Figure 4 further demonstrates dependency of misspecification levels for cases where assets' expected returns are lower than the expected return of the "intercept," and we must use the CML's "reflection" to identify the IV misspecification.



3 Empirical illustration

The following empirical examples demonstrate the IV misspecification existence.

We calibrate “true” and “incorrect” IV measures using the Fama–French 100 portfolios formed on size and book-to-market. The sample runs from October 1962 to June 2025. Table 1 reports, for each scenario below, the cross-sectional averages of four annualized measures—portfolio returns, time-series total volatility, true IV, and incorrect IV—and summary statistics of the true to misspecified IV ratio. At each month-end, IV measures are estimated from the previous 180 trading days.

[Table 1 is placed here]

We evaluate three market scenarios. In Scenario I, the zero-beta expected return is equal to the T-bill rate. In Scenario II, the expected market return is equal to the average value-weighted return on all CRSP common stocks over the previous six months. In Scenario III, the

monthly expected market return is 1% higher than the expected return of the Global Minimum Variance Portfolio. These configurations are typical examples of the sensitivities of misspecified IV measures to market parameters.

Table 1 shows that the level and dispersion of the true to misspecified IV ratio are highly sensitive to these parameters (market characteristics). Under Scenarios I and II, the true to misspecified IV ratio deviates substantially from one: average true to misspecified IV ratios are 78.4% and 77.0% of the misspecified IV, respectively. Ratios, however, vary widely: [43%, 100%] in Scenario I and [36%, 100%] in Scenario II. In contrast, Scenario III shows bias can be largely attenuated under alternative market assumptions: the average true to misspecified IV ratio is 97.5% with a median of 98.8%.

Finally, for each scenario, we regress the true to misspecified IV ratio to illustrate the effect of misspecification on empirical studies. As Section 2 shows, the ratio theoretically lies on $[0,1]$ with no systematic relation to true IV. However, the estimated coefficients are positive and statistically significant in all three scenarios, yet vary widely across them: a one-standard-deviation increase in true IV raises the ratio by 2.5%, 2.6%, and 0.3% (3.2%, 3.3%, and 0.3% of the average true to misspecified IV ratio) in Scenarios I, II, and III, respectively. This wide dispersion across scenarios underscores that the misspecification can produce seemingly meaningful estimates when no stable underlying economic relation exists.

Overall, this empirical illustration demonstrates that true to misspecified IV ratios are sensitive to stocks' parameters. These parameters strongly affect ratios of true to misspecified IV, and their cross-sectional dispersion.

4 Conclusion

We provide an IV identification in a world with no zero-variance return asset and demonstrate that the literature's identification of idiosyncratic volatility is misspecified. In reality, practically all risky assets include nondiversifiable zero-beta systematic volatility,

which the literature classifies as IV. The IV misspecification is nonlinear and nonmonotonic in all stock and market portfolio's attributes, and ranges from zero (with measure zero) to infinity. Our correct IV identification facilitates restudying IV-related questions, including the IV puzzle.

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Table 1. Calibrations of Fama-French 100 Size-BM portfolios

This table reports the effect of IV misspecification in the Fama–French 100 portfolios sorted by size and book-to-market. The sample covers October 1962 to June 2025. For three market scenarios, the first four rows show the annualized means of portfolio returns, time-series total volatility, true IV, and incorrect IV. The next seven rows show the mean, standard deviation, median, minimum and maximum, and upper and lower quartiles (75th and 25th percentiles) of the true to misspecified IV ratio. The following row reports the coefficient estimates and standard errors (in parenthesis) from regressing IV ratio on standardized True IV. *** indicates statistical significance at 1%. The last row presents the number of observations. Among 75,300 portfolio-month observations, 283 portfolio-months are dropped due to the lack of data for calculating IVs.

	Scenario I	Scenario II	Scenario III
Return (%)	13.3	13.3	13.3
Volatility (%)	21.6	21.6	21.6
True IV (%)	13.6	13.6	13.6
Incorrect IV (%)	17.2	15.2	17.4
Mean	78.4	77.0	97.5
SD	8.6	8.4	3.5
Min	42.8	36.0	56.7
IV Ratio (%) P25	72.9	71.8	96.6
Median	78.9	77.5	98.8
P75	84.5	82.8	99.7
Max	100	100	100
$\hat{\beta}$ (regress IV ratio on True IV)	2.487*** (0.030)	2.566*** (0.029)	0.310*** (0.013)
No. of Observations	75017	75017	75017